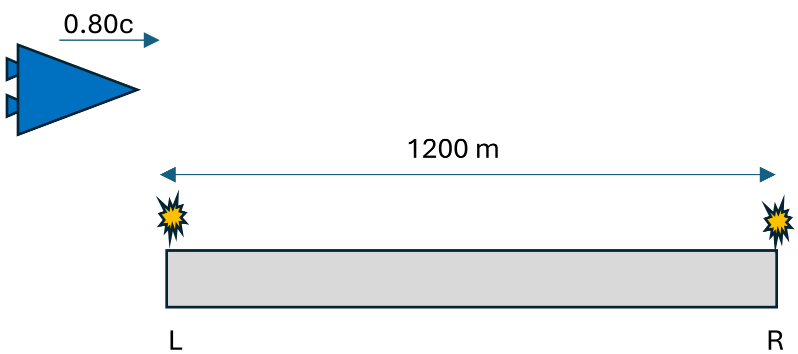
Teacher notes
Topic A

A specimen paper question on Relativity.

A specimen paper question asks: a rocket moves at 0.80c relative to a space station of proper length 1200 m. Two pulses are emitted from the Left (L) and Right (R) ends of the space station. The pulses are emitted at the same time according to space station observers.



What is the distance between the pulses according to the rocket?

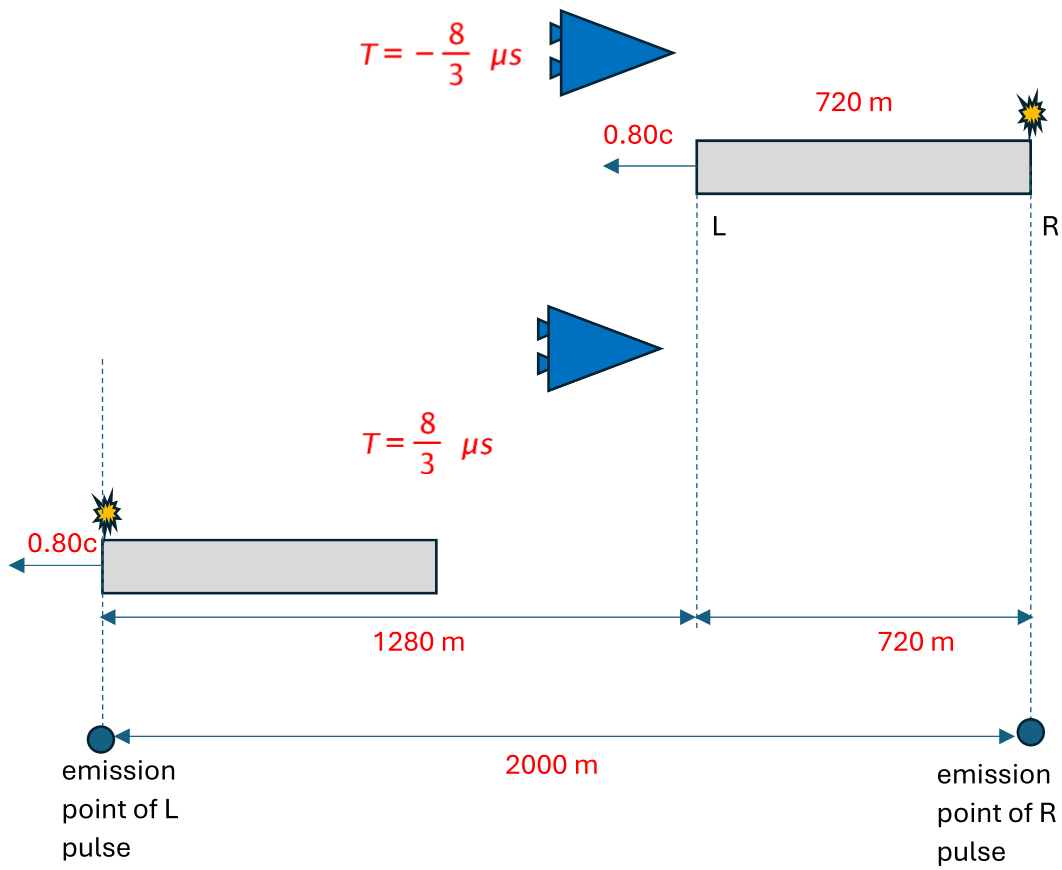
The question is really asking about the distance between the points at which the pulses were emitted.

This is a different question from asking about the length of the space station according to the rocket. To find the length of the space station the rocket must measure the position of the ends of the space station *at the same time in the rocket frame*. The pulses are not emitted at the same time according to the rocket, so the question is not equivalent to asking about the length of the space station.

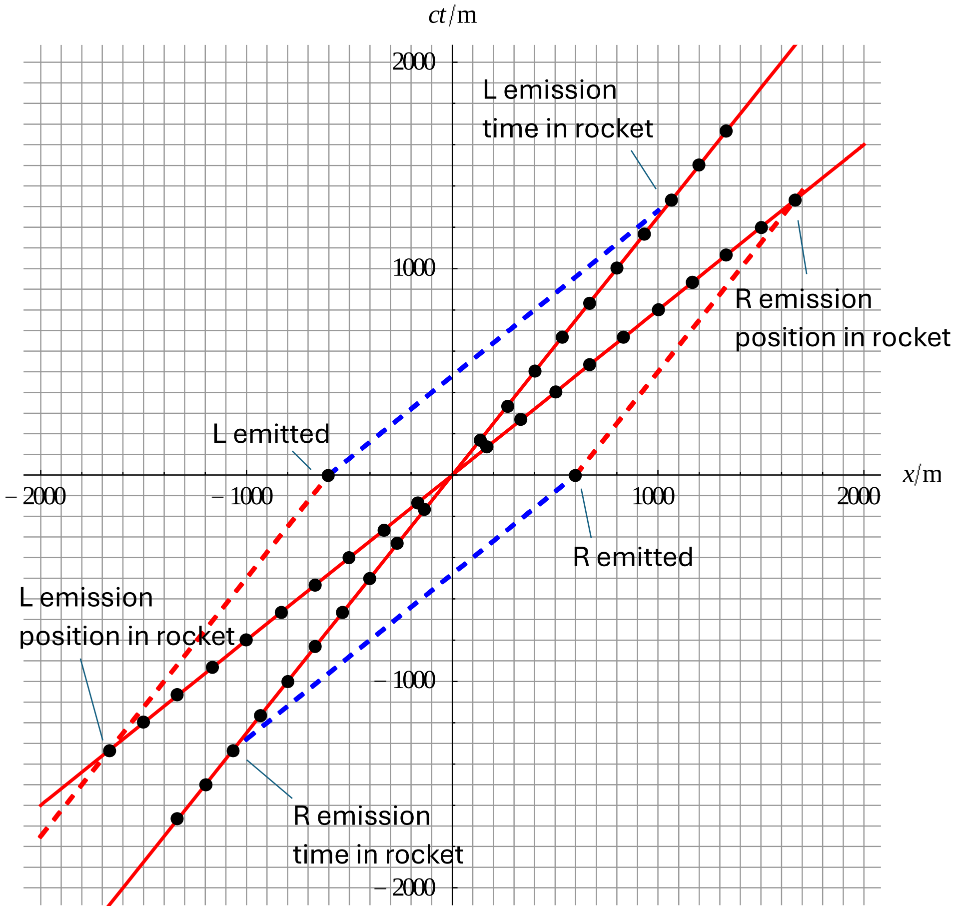
Once we understand this, the answer is simply given by a Lorentz transformation. In the space station frame, $\Delta x = x_R - x_L = 1200$ m and $\Delta t = 0$. Hence, $\Delta x' = \gamma(\Delta x - v\Delta t) = \frac{5}{3} \times (1200 - 0) = 2000$ m.

We can understand this in this way: the rocket observers think of themselves at rest and the space station is moving to the left with speed 0.80c relative to the rocket. The right pulse is emitted first in the rocket frame. When the right pulse is emitted the left end is a distance equal to the length of the space station away. This length is the Lorentz contracted length of $\frac{1200}{\frac{5}{3}} = 720$ m. In the time it takes the L pulse to be emitted, the space station moves a distance to the left equal to $0.80c\times\Delta t'$ where $\Delta t'$ is the time between emissions according to the rocket. This can be found from another Lorentz transformation: $\Delta t'=\gamma\left(\Delta t-\frac{v}{c^2}\Delta x\right)=\frac{5}{3}\times\left(0-\frac{0.80c}{c^2}\times 1200\right)=-\frac{16}{3}\times 10^{-6}\text{ s}$. The minus sign indicates that the right pulse was emitted first. Hence the distance travelled in this time is $\frac{16}{3}\times 10^{-6}\times 0.80\times 3\times 10^{8}=1280\text{ m}$. Hence the distance between the points where the pulses were emitted is $720+1280=2000\text{ m}$ in agreement with the original answer.

The sequence of events is shown below from the point of view of the rocket. The times and distances in red are rocket frame values.

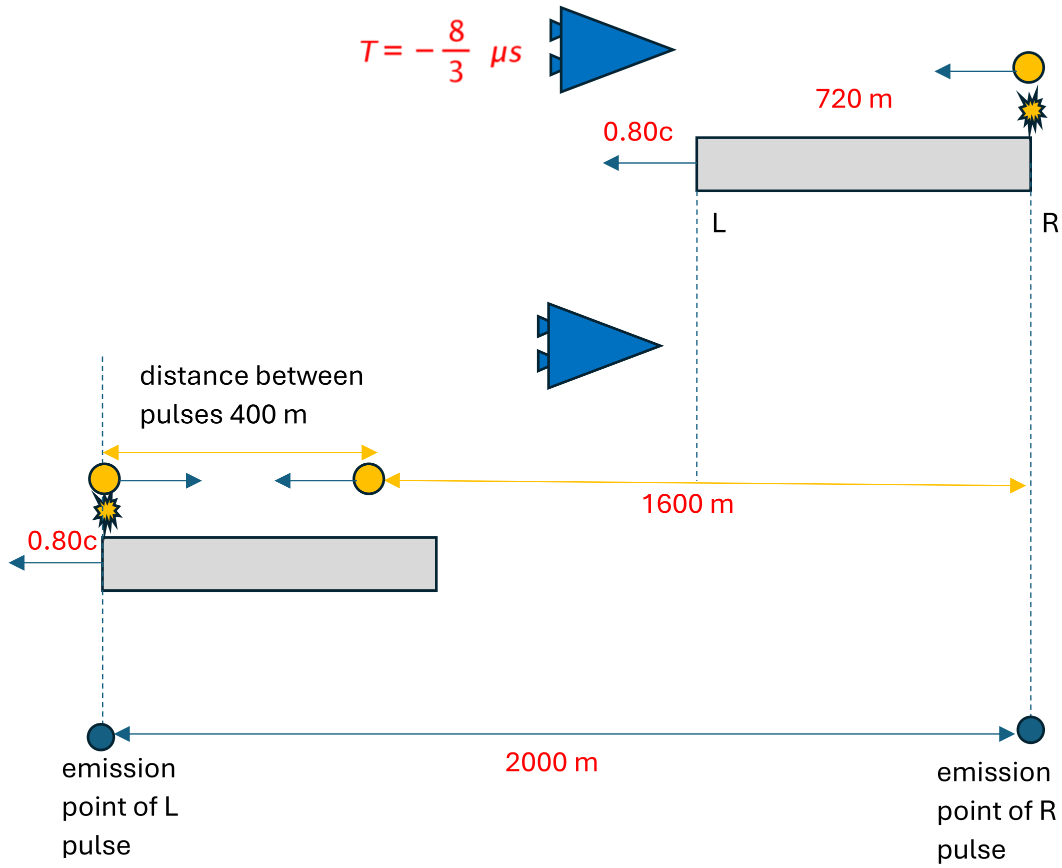


This is shown on the spacetime diagram below. The dots on the rocket axes are 100 m apart. The diagram shows the lamps going on at the same time in the space station frame. The emission points have coordinates (in the rocket frame) found by the intersection of the red dotted lines (parallel to the time axis) with the rocket space axis. These points are 2000 m apart. The times in the rocket frame are given by the intersection of the blue dotted lines (parallel to the rocket space axis) with the rocket time axis. The emission points are 1600 m apart (timewise) so the time in between emissions is $ct=1600\Rightarrow t=\frac{1600}{c}=\frac{16}{3}$ μs as we found before.



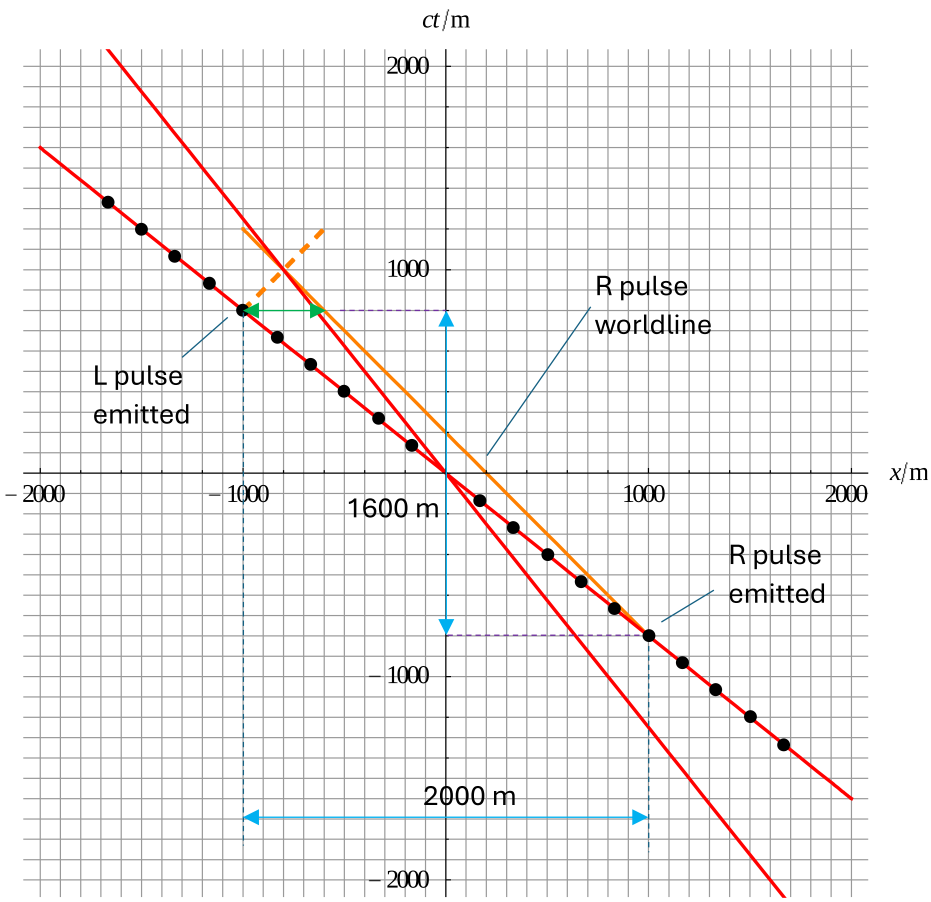
PS

Notice that when the L pulse is emitted the R pulse has travelled a distance of $\frac{16}{3}\times 10^{-6}\times 3\times 10^{8}=1600\text{ m}$. So, technically, when the question asked about *the distance between the pulses*, the answer should have been $2000-1600=400\text{ m}$. (Assuming they wanted the distance between the pulses the instant the L pulse was emitted!)



We can see all this on a spacetime diagram. This time we choose to treat the rocket at rest and the space station moving to the left. So, the red axes are the space station axes. We see again that the distance between the points of emission are 2000 m (blue double-headed arrow). When L is emitted, the R pulse has already moved to the left. We need to find the distance between the pulses at the time of the emission of L. This is the green double-headed arrow. It is 4 squares long, i.e. 400 m.

(Incidentally, we can clearly see that the L signal is emitted later than R by an amount ct = 1600 m, i.e. $t = \dfrac{1600}{3 \times 10^8} = \dfrac{16}{3}$ μs as we found earlier.)



To make matters worse, we were never told that the R pulse goes to the left! If it went to the right then the distance between the pulses would be 2000 + 1600 = 3600 m!

This question is totally ambiguous:

- It does not make clear the difference between distance between pulses and distance between points of emission
- It does not specify the direction of the pulses
- It does not specify the time at which the distance between the pulses is to be determined

So the answer can be 2000 m, 400 m, 3600 m or in fact any distance from zero to infinity!